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147. Proposed by R. A. WELLS, Professor of Mathematics, Franklin College, Athens, Ohio.

Find the locus in space of the point which is equally illuminated by each of two unequal lights whose intensities are a and b ($a > b$), placed at a distance c from each other.

Solution by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let (x, y, z) be the rectangular coördinates of any one of the illuminated points, the mid-point of the line joining the two lights taken along the x -axis as origin; then the coördinates of the two lights may be given as $(l, 0, 0)$, $(-l, 0, 0)$, and by theory

$$\frac{a}{(x-l)^2 + y^2 + z^2} = \frac{b}{(x+l)^2 + y^2 + z^2} \dots (1),$$

$$\text{or } x^2 + y^2 + z^2 + 2l \frac{a+b}{a-b} x + l^2 = 0 \dots (2), \text{ a sphere.}$$

Also solved by G. B. M. ZERR, and J. SCHEFFER.

148. Proposed by DR. E. D. ROE, JR., Associate Professor of Mathematics in Syracuse University, Syracuse, N. Y.

The condition that two triangles, abc, xyz , are similar is

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ x & y & z \end{vmatrix} = 0,$$

and the condition that the triangle abc is equilateral is

$$\begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix} = 0.$$

(Used in solving 130.)

I. Solution by the PROPOSER.

First. *The given conditions are necessary*, for if two triangles abc, xyz , be similar, then $\frac{a-b}{x-y} = \frac{b-c}{y-z} = \frac{c-a}{z-x} = r \dots (1)$, for this expresses not only that the homologous sides are proportional, but also that the homologous angles are equal. (1) is equivalent to the equations

$$\begin{aligned} a-b &= r(x-y) \\ b-c &= r(y-z) \quad \dots (2). \\ c-a &= r(z-x) \end{aligned}$$

Multiplying the equations (2) by x, y , and z , respectively, and adding, we get

$$\Sigma(a-b)z = r \Sigma(x-y)z = 0 \dots (3),$$

or $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ x & y & z \end{vmatrix} = 0 \dots (4)$, which is therefore necessary at least, if the triangles are similar.

If the triangle abc is equilateral, it is also equiangular, and

$$\frac{a-b}{b-c} = \frac{b-c}{c-a} = \frac{c-a}{a-b} = r \dots (5), \text{ or}$$

$$\begin{aligned} a-b &= r(b-c) \\ b-c &= r(c-a) \quad \dots (6). \\ c-a &= r(a-b) \end{aligned}$$

Multiplying these equations by a, b, c , respectively, and adding, we have

$$\Sigma a^2 - \Sigma ab = r \Sigma (b-c)a = 0 \dots (7), \text{ or}$$

$$\begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix} = 0 \dots (8),$$

and this condition is also at least necessary.

Second. *These conditions are also sufficient.* For, with

$$\frac{a-b}{x-y} = r_1, \quad \frac{b-c}{y-z} = r_2, \quad \frac{c-a}{z-x} = r_3 \dots (9),$$

or what is the same thing, with

$$\begin{aligned} a-b &= r_1(x-y) \\ b-c &= r_2(y-z) \quad \dots (10) \\ c-a &= r_3(z-x) \end{aligned}$$

$$\text{form } \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ x & y & z \end{vmatrix} = (a-b)z + (b-c)x + (c-a)y$$

$$= r_1 z(x-y) + r_2 x(y-z) + r_3 y(z-x) \dots (11),$$

and put this equal to zero. We are then to prove in fact that this obliges r_1, r_2, r_3 to be equal to one another. We have

$$r_1 z(x-y) + r_2 x(y-z) + r_3 y(z-x) = 0 \dots (12).$$

$$r_1(x-y) + r_2(y-z) + r_3(z-x) = 0 \dots (13).$$

The latter equation comes by adding equations (10) together. By solving (12) and (13),

$$\begin{aligned}
& \frac{r_1 \begin{vmatrix} x(y-z) & y(z-x) \\ (y-z) & (z-x) \end{vmatrix}}{r_3 \begin{vmatrix} z(x-y) & x(y-z) \\ (x-y) & (y-z) \end{vmatrix}} = \frac{(z-x) \begin{vmatrix} x & y \\ 1 & 1 \end{vmatrix}}{(x-y) \begin{vmatrix} z & x \\ 1 & 1 \end{vmatrix}} = 1. \\
& \dots (14). \\
& \frac{r_2 \begin{vmatrix} y(z-x) & z(x-y) \\ (z-x) & (x-y) \end{vmatrix}}{r_3 \begin{vmatrix} z(x-y) & x(y-z) \\ (x-y) & (y-z) \end{vmatrix}} = \frac{(z-x) \begin{vmatrix} y & z \\ 1 & 1 \end{vmatrix}}{(y-z) \begin{vmatrix} z & x \\ 1 & 1 \end{vmatrix}} = 1.
\end{aligned}$$

or $r_1=r_3$, $r_2=r_3$, and therefore $r_1=r_2=r_3$. . (15), whence the homologous sides of the triangles abc , xyz are proportional, and their homologous angles are equal, and the triangles are in fact similar. In the other case form

$$\begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix} = r_1 a(b-c) + r_2 b(-a) + r_3 c(a-b)$$

where $r_1 = \frac{a-b}{b-c}$, $r_2 = \frac{b-c}{c-a}$, $r_3 = \frac{c-a}{a-b}$. . . (16),

and put this equal to zero. We have

$$\begin{aligned}
& r_1 a(b-c) + r_2 b(c-a) + r_3 c(a-b) = 0. \\
& r_1 (b-c) + r_2 (c-a) + r_3 (a-b) = 0. \dots (17). \\
& r_1 r_2 r_3 = 1.
\end{aligned}$$

The last two of equations (17) come from (16). As before, by solving the first two equations of (17) for r_1/r_3 , r_2/r_3 , we get $r_1=r_2=r_3=r$ say; by the last of equation (17) $r^3=1$, therefore

$$\begin{aligned}
& r=1 \\
& r=\cos 60^\circ + i \sin 60^\circ \dots (18), \\
& r=\cos 120^\circ + i \sin 120^\circ
\end{aligned}$$

and the only solution applying to a geometrical triangle in the plane is the second, which in fact determines an equilateral triangle, since modulus $r=1$, argument $r=60^\circ$.

Since the conditions are both necessary and sufficient, they are the required conditions.

II. Solution by LON C. WALKER, Assistant in Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

Let z_1, z_2, z_3 represent the vertices of a triangle in a z -complex-number-

plane, and let z'_1, z'_2, z'_3 represent the corresponding vertices in a z' -complex-number-plane.

A transformation can always be found that will change z_1 in z'_1 , and z_2 into z'_2 . For this it is sufficient that a and b in

$$\left. \begin{aligned} z'_1 &= az + b \\ z'_2 &= az_2 + b \end{aligned} \right\} \dots (1),$$

shall satisfy the condition

$$a = \frac{z'_2 - z'_1}{z_2 - z_1}, \text{ and } b = \frac{z'_1 z_2 - z'_2 z_1}{z_2 - z_1}.$$

Every transformation of the plane into itself that leaves every figure in the plane similar to itself is expressible in the form

$$z' = az + b \dots (2).$$

The value of a and b in (1) must satisfy the equation $z'_3 = az_3 + b$. For this the necessary and sufficient condition is

$$\frac{z'_3 - z'_1}{z_3 - z_1} = \frac{z'_2 - z'_1}{z_2 - z_1}; \therefore \begin{vmatrix} z'_1 & z_1 & 1 \\ z'_2 & z_2 & 1 \\ z'_3 & z_3 & 1 \end{vmatrix} = 0.$$

The conditions that the sides shall be equal are

$$\frac{z_1 - z_2}{z_2 - z_3} = \frac{z_2 - z_3}{z_3 - z_1} = \frac{z_3 - z_1}{z_1 - z_2}; \therefore \begin{vmatrix} z_1 & z_2 & 1 \\ z_2 & z_3 & 1 \\ z_3 & z_1 & 1 \end{vmatrix} = 0.$$

Also demonstrated by WILLIAM HOOVER, G. B. M. ZERR, J. W. YOUNG, and J. SCHEFFER.

CALCULUS.

108. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

The hypotenuse of a plane right triangle increases uniformly at the rate of 1-12 of an inch a second. If the legs are as 2 to 3, at what rate is the area of the triangle increasing when the perpendicular from the right angle upon the hypotenuse is 12 inches?

Solution by C. HORNING, A. M., Heidelberg University, Tiffin, O.; H. C. WHITAKER, Ph. D., Manual Training School, Philadelphia, Pa.; J. W. YOUNG, Cornell University, Ithaca, N. Y.; P. S. BERG, Larimore, N. D.; and D. G. DORRANCE, Jr., Camden, N. Y.

Let x = the length of the hypotenuse, and y = the area.

Then $dx = \frac{1}{12}$, and dy is required when the perpendicular from the right angle to the hypotenuse equals 12.

From the condition of the problem $y = \frac{3x^2}{13}$, whence $dy = \frac{6xdx}{13}$.